



# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

**FIFTH SEMESTER – NOVEMBER 2023**

**UMT 5601 – GRAPH THEORY**

Date: 16-11-2023

Dept. No.

Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

## SECTION A - K1 (CO1)

**Answer ALL the Questions**

**(10 x 1 = 10)**

1. **Answer the following**

- a) When do you say that a graph is connected?
- b) Given an example of Euler graph, but not Hamiltonian.
- c) Define tree.
- d) What is edge connectivity of a graph?
- e) Define planar graph.

2. **Fill in the blanks**

- a) The degree of a pendant vertex is \_\_\_\_\_.
- b) The number of edges in a complete graph on six vertices is \_\_\_\_\_.
- c) The rank of a connected graph is \_\_\_\_\_.
- d) A graph with vertex connectivity one is called \_\_\_\_\_.
- e) The chromatic number of a circuit on five vertices is \_\_\_\_\_.

## SECTION A - K2 (CO1)

**Answer ALL the Questions**

**(10 x 1 =**

**10)**

3. **Choose the correct answer**

- a) The total sum of degrees of a connected graph with 15 vertices and 20 edges  
(i) 30    (ii) 35    (iii) 40    (iv) 50
- b) A closed walk that traverses all the edges of a graph  
(i) Euler    (ii) Gauss    (iii) Hamiltonian    (iv) Kuratowski
- c) The number of edges in a tree with 100 vertices  
(i) 100    (ii) 99    (iii) 98    (iv) 0
- d) A circuit formed by adding exactly one chord of a graph to the spanning tree  
(i) fundamental    (ii) minimal    (iii) maximal    (iv) none of these
- e) The indegree of a vertex in a digraph  
(i) number of incoming edges    (ii) number of outgoing edges  
(iii) number of nonadjacent edges    (iv) number of nonadjacent vertices

4.	<b>True or False</b>
a)	Konigsberg bridge problem deals with connecting 6 bridges.
b)	A connected graph having all the vertices with degree two is Hamiltonian.
c)	The number of non-isomorphic trees on three vertices is two.
d)	The edge connectivity of a graph cannot exceed the vertex connectivity.
e)	Planar graphs have crossing number one.
<b>SECTION B - K3 (CO2)</b>	
	<b>Answer any TWO of the following (2 x 10 = 20)</b>
5.	Show that a graph $G$ is disconnected if and only if its vertex set $V$ can be partitioned into two nonempty disjoint subsets $V_1$ and $V_2$ such that there exists no edge in $G$ whose one end vertex is in $V_1$ and the other end in $V_2$ .
6.	Prove that a connected graph $G$ is an Euler graph if and only if it can be decomposed into circuits.
7.	Establish that every tree has either one or two centers.
8.	Show that every circuit has an even number of edges in common with any cut-set
<b>SECTION C – K4 (CO3)</b>	
	<b>Answer any TWO of the following (2 x 10 = 20)</b>
9.	Analyze that a tree with $n$ vertices has $n - 1$ edges.
10.	Examine that if $n$ is an odd number and $n \geq 3$ , there are $(n - 1)/2$ edge-disjoint Hamiltonian circuits in a complete graph on $n$ vertices.
11.	Explain any three basic operations on graphs.
12.	Derive that a graph of $n$ vertices is a complete graph if and only if its chromatic polynomial is given by $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1)$ .
<b>SECTION D – K5 (CO4)</b>	
	<b>Answer any ONE of the following (1 x 20 = 20)</b>
13.	(a) With respect to a given spanning tree $T$ , exhibit that a chord $c_i$ determines a fundamental circuit $\Gamma$ which occurs in every fundamental cut-set associated with the branches in $T$ and in no others. (b) Evaluate that a simple graph with $n$ vertices and $k$ components can have at most $(n - k)(n - k + 1)/2$ edges.
14.	(a) Determine that the complete graph on five vertices is nonplanar. (b) Evaluate that every tree with two or more vertices is 2-chromatic.
<b>SECTION E – K6 (CO5)</b>	
	<b>Answer any ONE of the following (1 x 20 = 20)</b>
15.	(a) Formulate that a given connected graph $G$ is an Euler graph if and only if all vertices of $G$ are of even degree. (b) Develop a formula to find the number of regions in a connected planar graph with $n$ vertices and $e$ edges.
16.	(a) Construct the ring sum of any two cut-sets in a connected graph and identify that whether it is a third cut-set or an edge disjoint union of cut-sets. (b) Represent a real-life situation as a weighted connected graph and extract a shortest spanning tree.

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